CBSE Board
Class X Mathematics
Sample Paper 4 (Standard)

Time: 3 hrs **Total Marks: 80**

General Instructions:

1. This question paper contains **two parts** A and B.
2. Both **Part A** and **Part B** have internal choices.

Part – A:

1. It consists **two sections** - I and II.
2. **Section I** has **16 questions** of **1 mark** each. Internal choice is provided in **5 questions**.
3. **Section II** has **4 questions** on **case study**. Each case study has **5 case-based sub-parts**. An examinee is to attempt any **4 out of 5 sub-parts**. Each subpart carries **1 mark.**

Part – B:

1. It consists **three sections** – III, IV and V
2. **Section III: Question No 21 to 26** are **Very short answer** Type questions of **2 marks** each.
3. **Section IV: Question No 27 to 33** are **Short Answer Type** questions of **3 marks** each.
4. **Section V: Question No 34 to 36** are **Long Answer Type** questions of **5 marks** each.
5. Internal choice is provided in **2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.**

Part A

Section I

Section I has 16 questions of 1 mark each.

(Internal choice is provided in 5 questions)

1. The decimal expansion of the rational number $\frac{2^3}{2^2.5}$ will terminate after how many decimal places?

OR

What is an irrational number

2. A letter is chosen at random from the word "PROBABILITY". Find the probability that it is a vowel.

3. If the system of equations $2x + 3y = 5$, $4x + ky = 10$ has infinitely many solutions, then find the value of.

OR

If the point (3, a) lies on the line represented by $2x - 3y = 5$ then find the value of a.

4. Find the value of $\frac{2\tan 30°}{1 - \tan^2 30°}$ =

5. What is the value of $\tan\theta$, if θ and $2\theta - 45°$ are acute angles such that $\sin\theta = \cos(2\theta - 45°)$?

6. Find the mid-point of the line segment joining P(−2, 8) and Q(−6, −4).

OR

Find the value of x, for which the points (x, −1), (2, 1) and (4, 5) lie on a line.

7. The ordinate of a point is twice its abscissa. If its distance from the point (4, 3) is $\sqrt{10}$, then the find the coordinates of the point.

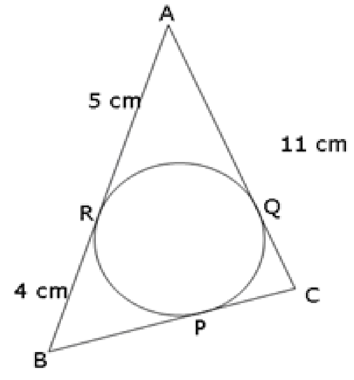
8. What is the maximum volume of a cone that can be carved out of a solid hemisphere of radius r?

9. If the sum of the zeros of the polynomial $f(x) = 2x^3 - 3kx^2 + 4x - 5$ is 6, then find the value of k.

10. If $\Delta ABC \sim \Delta DEF$. If BC = 3 cm, EF = 4 cm and ar(ΔABC) = 54 cm^2 then find ar(ΔDEF).

OR

In the given figure, AR = 5cm, BR = 4cm and AC =11cm. What is the length of BC?



11. What is the value of 10^{th} term of an A.P., if its first term is p and common difference is q?

OR

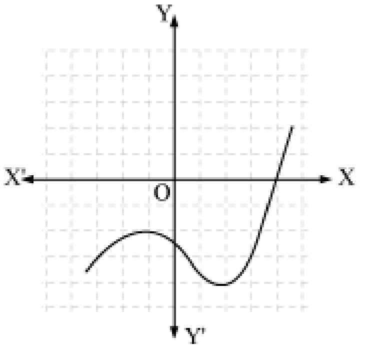
Find the value of x for which 2x, x + 10, and 3x + 2 are in A.P.

12. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively: $\frac{1}{4}$ and -1

13. What is the LCM of $2^3 \times 3 \times 5$ and $2^4 \times 5 \times 7$?

14. Find the condition on k if the equation $x^2 + 4x + k = 0$ has real and distinct roots.

15. The graph of $y = p(x)$ is given in the following figure for some polynomial $p(x)$. What is/are the number of zeroes of $p(x)$?



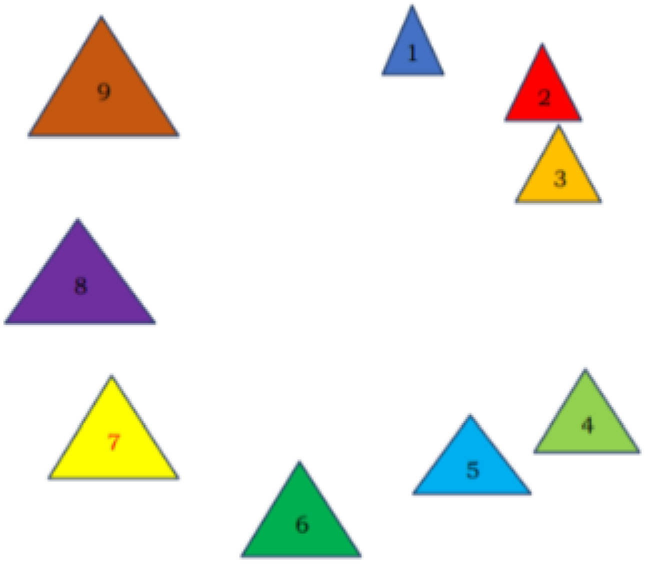
16. Determine the set of values of p for which the quadratic equation $px^2 + 6x + 1 = 0$ has real roots.

Section II

(Q 17 to Q 20 carry 4 marks each)

Case study based questions are compulsory. Attempt any four sub parts of each question. Each subpart carries 1 mark

17. Case Study Based- 1
Numbers and Equilateral Triangles.



Rahul and Sunil were feeling bored during the lockdown. They both created a number game. Sunil prepared nine equilateral triangles and numbered them from 1 to 9. The numbers written on the triangles also represent length of each side of the triangle (in cm).
Sunil arranged them in the form of a circle. He asked Rahul to remove alternate triangles starting from number 1, going clockwise, until only one triangle remained.

(a) The triangle which Rahul removed in the first round are in order, numbered 1, 3, 5, 7, 9. If Rahul continues in the same manner, which numbered triangle will be left in the last?
(i) 4
(ii) 2
(iii) 8
(iv) 6

(b) In the second round, Sunil started counting with triangle numbered 1 and eliminated every third triangle, until only one triangle remained. Which of the following triangle will be left in the end?
Triangle number:
(i) 1
(ii) 3
(iii) 7
(iv) 6

(c) Rahul added two more triangles in the circle and numbered these as triangle 10 and triangle 11. In this round, Rahul started counting with triangle numbered 1, but anticlockwise, and eiminated every fifth triangle, until only one triangle remained. Which triangle will be left in the end?
Triangle number:
(i) 2
(ii) 4
(iii) 5
(iv) 8

(d) f there are 9 triangles, will the perimeters of the triangles follow any pattern? If so, write the pattern?
(i) They are multiple of 3.
(ii) They are multiple of 6.
(iii) They are multiple of 2.
(iv) They are multiple of 4.

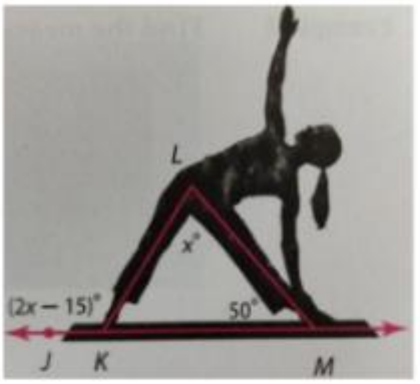
(e) Are the areas of the triangles numbered 3, 4 and 6, 8 in proportion? If yes then write down the ratio.
(i) 9: 16
(ii) 3: 4
(iii) 7: 8
(iv) 16: 9

18. Case Study based-2

Types of angles and angle sum property of a triangle

It is 7:00 am!

Shikha rolls out her yoga mat and starts her warm up session with stretching and bending. Anaya her daughter is sitting nearby, observing her mother's daily ritual. Anaya takes a picture of her mother while she was in a yoga posture and label it as shown.



(a) Angles ∠LKM and ∠JKL are called as?
   (i) Linear Pair of angles
   (ii) Vertically opposite angles
   (iii) Complementary angles
   (iv) Corresponding angles

(b) Find m∠LKM.
   (i) 195° – x
   (ii) 185° – 2x
   (iii) 195° – 2x
   (iv) 185° – x

(c) Find m∠KLM.
   (i) 115°
   (ii) 65°
   (iii) 50°
   (iv) 180°

(d) Which of the following is true for △LKM?
   (i) △LKM is an equilateral triangle.
   (ii) △LKM is an isosceles triangle.
   (iii) △LKM is a right angle triangle.
   (iv) All of the above

(e) What is the measurement of the ∠LKJ?
   (i) 115°
   (ii) 65°
   (iii) 50°
   (iv) 180°

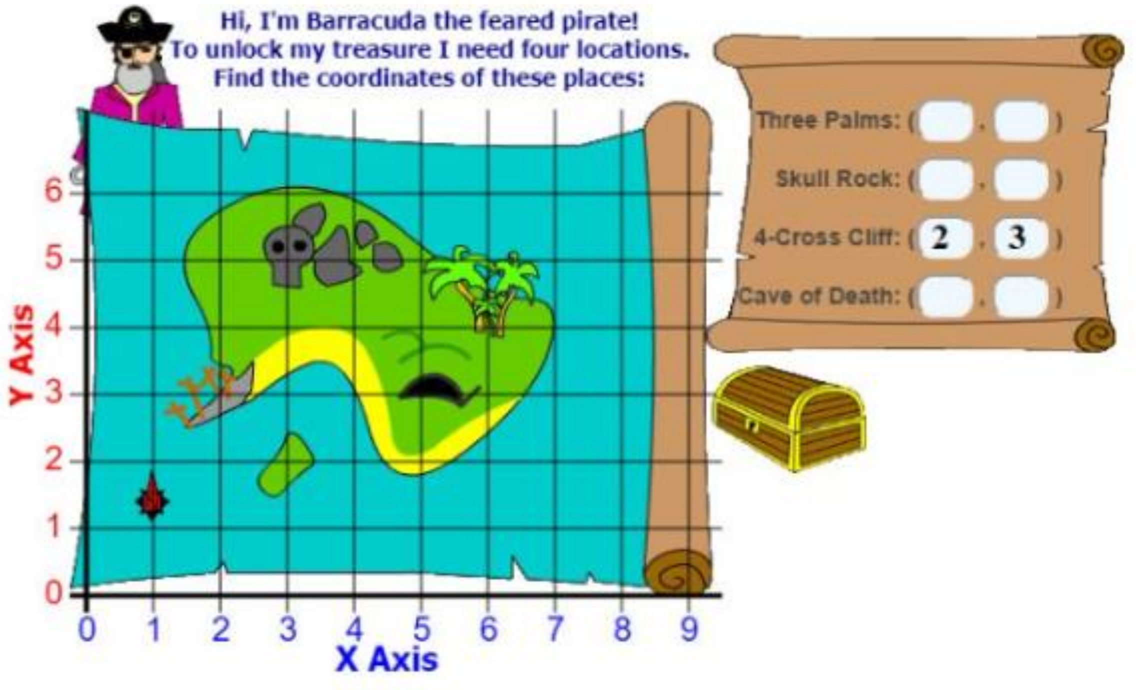
19. Case Study Based- 3
THE TREASURE ISLAND

Understanding Graphs:

On the graph sheet, a point is located using a pair of numbers such as (x, y)

- The first number 'x' shows the horizontal distance of the point (i. e left or right) on the horizontal line.
- The second number 'y' shows the vertical distance of the point (i. e up of down) right) on the vertical line.
- The point where X – axis and Y – axis cross each other at 90° called the Origin denoted by (0, 0).
- Clearly the X – axis and Y – axis divide the plane is known as Cartesian plane.
- We measure everything on the Cartesian plane with respect to Origin.

Rita and Renu are playing a board game of Treasure Island.

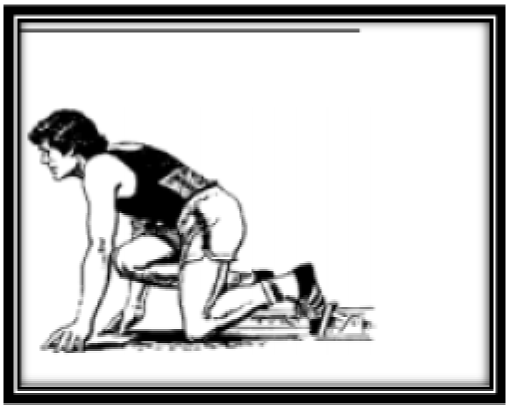


(a) The coordinate of CAVE of DEATH
(i) (3, 5)
(ii) (3, 3)
(iii) (5, 5)
(iv) (5, 3)

(b) The coordinate of THREE PALMS
(i) (6, 3)
(ii) (3, 6)
(iii) (5, 2)
(iv) (9, 5)

(c) The distance between FOUR CROSS CLIFF and the CAVE of DEATH is
(i) 3 units
(ii) 5 units
(iii) 2 units
(iv) None of these
(d) What is the distance of SKULL ROCK from x – axis?
(i) 3 units
(ii) 5 units
(iii) 2 units
(iv) None of the
(e) The mid – point of CAVE of DEATH and THREE PALMS is
(i) (5.5, 3.5)
(ii) (5, 3)
(iii) (3.5, 5.5)
(iv) (3, 5)

20. Case Study Based- 4
110m RACE
A stopwatch was used to find the time that it took a group of students to run 110m.



Time(in sec)	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120
No. of students	7	10	15	5	3

(a) Estimate the mean time taken by a student to finish the race.
(i) 54.6
(ii) 63.5
(iii) 43.5
(iv) 50.5
(b) What will be the lower limit of the modal class?
(i) 20
(ii) 40

(iii) 60
(iv) 80

(c) Which of the following are measures of Central Tendency?
(i) Mean
(ii) Median
(iii) Mode
(iv) All of the above

(d) The sum of upper limits of median class and modal class is
(i) 60
(ii) 120
(iii) 80
(iv) 160

(e) How many students finished the race within 1 min?
(i) 18
(ii) 37
(iii) 17
(iv) 8

Part B

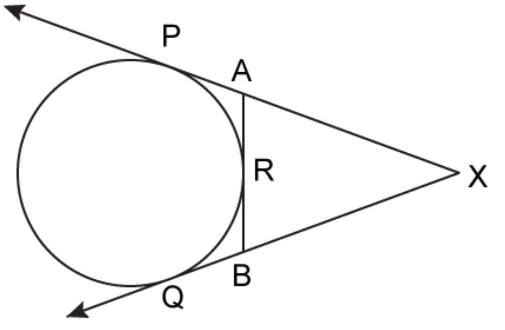
All questions are compulsory. In case of internal choices, attempt any one.

Section III

(Q 21 to Q 26 carry 2 marks each)

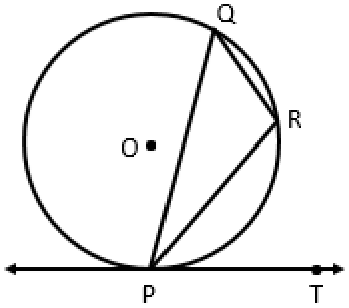
21. Show that any number of the form 4^n, $n \in N$ can never end with the digit 0.

22. In the given figure, XP and XQ are tangents from X to the circle. R is a point on the circle. Prove that XA + AR = XB + BR.



OR

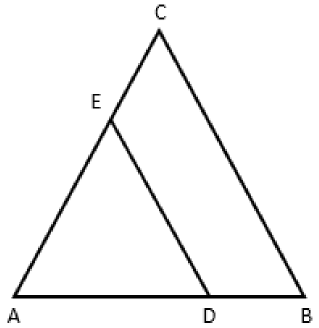
In the figure, PQ is a chord of a circle with centre O and PT is a tangent. If ∠QPT = 60°, find ∠PRQ .



23. A bag contains 3 red balls and 5 black balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is (i) red (ii) not red?

24. A cylinder and a cone have bases of equal radii and are of equal heights. Show that their volumes are in the ratio of 3:1.

25. In the adjoining figure, DE is parallel to BC. If AD = x, DB = x – 2, AE = x + 2 and EC = x – 1, find the value of x.



OR

Corresponding sides of two triangles are in the ratio 2:3. If the area of the smaller triangle is 48 cm^2, determine the area of the larger triangle.

26. A solid metal cone with radius of base 12 cm and height 24 cm is melted to form solid spherical balls of diameter 6 cm each. Find the number of balls thus formed.

Section IV

(Q 27 to Q 33 carry 3 marks each)

27. Prove that: $\frac{\sec A + \tan A}{\sec A - \tan A} = \left(\frac{1 + \sin A}{\cos A}\right)^2$

OR

Without using tables evaluate:

$$\left(\frac{\sin 47^\circ}{\cos 43^\circ}\right)^2 + \left(\frac{\cos 43^\circ}{\sin 47^\circ}\right)^2 - 4\cos^2 45^\circ$$

28. Solve for x and y:

$\frac{x}{a} + \frac{y}{b} = 2;\qquad ax - by = a^2 - b^2$

29. Prove that $\frac{3}{2\sqrt{5}}$ is an irrational number.

OR

Find the HCF of 96 and 404 by prime factorisation method. Hence, find their LCM.

30. Cards numbered from 1 to 18 are put in a box and mixed thoroughly. One card is drawn at a random. Find the probability that the card drawn bears:

i. a prime number
ii. a factor of 18
iii. a number divisible by 2 and 3

31. The 14th term of an A.P. is twice its 8th term. If its 6th term is –8, then find the sum of its first 20 terms.

32. Find all zeros of the polynomial $(2x^4 - 9x^3 + 5x^2 + 3x - 1)$ if two of its zeros are $(2 + \sqrt{3})$ and $(2 - \sqrt{3})$.

33. Find the area of the minor segment of a circle of radius 14 cm, when its central angle is 60°. Also find the area of the corresponding major segment. [Use $\pi = \frac{22}{7}$]

Section V

(Q 34 to Q 36 carry 5 marks each)

34. Construct a triangle similar to ΔABC in which AB = 4.6 cm, BC = 5.1 cm, m∠A = 60° with scale factor 4: 5.

35. A man in a boat rowing away from a light house 100 m high, takes 2 minutes to change the angle of elevation of the top of the light house from 60° to 45°. Show that the speed of the boat is $50\left(\frac{3-\sqrt{3}}{3}\right)$ m/min.

36. By increasing the list price of a book by Rs. 10, a person can buy 10 less books for Rs. 1200. Find the original list price of the book.

OR

A motor boat, whose speed is 15km/ hr in still water, goes 30 km downstream and comes back in a total time of 4hrs 30mins. Find the speed of the stream.

CBSE Board
Class X Mathematics
Sample Paper 4 (Standard) – Solution

Part A

Section I

1. If the denominator of a rational number is of the form 2^n5^m, then it will terminate after n places if n>m or m places if m>n.

Now, $\frac{2^3}{2^25}=\frac{2}{5}=\frac{2}{2^\circ 5}$ will terminator after 1 decimal place.

OR

Any number that can't be expressed in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$ is called an irrational number.

Examples: $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{5}$.

2. In the word "PROBABILITY", there are 11 letters out of which 4 are vowels (O, A, I, I).

P(getting a vowel) = $\frac{4}{11}$

3. $2x + 3y = 5$, $4x + ky = 10$

$a_1 = 2$, $b_1 = 3$, $a_2 = 4$ and $b_2 = k$

Conditions for infinitely many solutions is

$$\frac{a_1}{a_2}=\frac{b_1}{b_2} \Rightarrow \frac{2}{4}=\frac{3}{k}=\frac{1}{2} \Rightarrow k = 6$$

OR

The point (3, a) lies on the line $2x - 3y = 5$.

Substituting the values of x and y in the given equation:

$$2 \times 3 - 3 \times a = 5 \Longrightarrow 6 - 3a = 5 \Longrightarrow 3a = 1 \Rightarrow a = \frac{1}{3}$$

4. $$\frac{2\tan 30^\circ}{1-\tan^2 30^\circ}=\frac{2\times\frac{1}{\sqrt{3}}}{1-\left(\frac{1}{\sqrt{3}}\right)^2}=\frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}}=\frac{2}{\sqrt{3}}\times\frac{3}{2}=\sqrt{3}$$

Also, $\tan 60^\circ = \sqrt{3}$

5. $\sin\theta = \cos(2\theta - 45^\circ)$

$\Rightarrow \cos(90^\circ - \theta) = \cos(2\theta - 45^\circ) \Rightarrow 90^\circ - \theta = 2\theta - 45^\circ \Rightarrow 3\theta = 135^\circ \Rightarrow \theta = 45^\circ$

$\Rightarrow \tan 45^\circ = 1$

6. Let R be the mid-point of PQ, then, the coordinates of mid-point of PQ, i.e., R are $\left[\frac{(-2-6)}{2}, \frac{(8-4)}{2}\right]$ = (–4, 2)

OR

Area of a triangle = 0

$\Rightarrow \frac{1}{2}\left|x(1-5)+2(5+1)+4(-1-1)\right|=0$

$\Rightarrow \frac{1}{2}\left|-4x+12-8\right|=0$

$\Rightarrow x=1$

7. Let the coordinates of the point be P(x, 2x). Let Q be the point (4, 3).

$PQ^2 = (4-x)^2 + (3-2x)^2 = 10$

$16 + x^2 - 8x + 9 + 4x^2 - 12x = 10$

$\Rightarrow 5x^2 - 20x + 15 = 0$

$\Rightarrow x^2 - 4x + 3 = 0$

$\Rightarrow (x-3)(x-1) = 0$

$\Rightarrow x = 1$ or $x = 3$

So, 2x = 2 or 6

Hence, the coordinates of the required point are (1, 2) or (3, 6).

8. The maximum volume of a cone that can be carved out of a solid hemisphere of radius r

is $\frac{\pi r^3}{3}$.

9. $f(x) = 2x^3 - 3kx^2 + 4x - 5$

a = 2, b = –3k, c = 4 and d = –5

Let α, β, γ be the zeros of the given polynomial.

$\alpha+\beta+\gamma = \frac{-b}{a} = \frac{3k}{2}$

$\frac{3k}{2} = 6 \Rightarrow k = 4$

10. ΔABC ~ ΔDEF, BC = 3 cm, EF = 4 cm and ar(ΔABC) = 54 cm^2

The ratio of the areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides.

$\Rightarrow \frac{BC^2}{EF^2} = \frac{54}{ar(\Delta DEF)} \Rightarrow \frac{3^2}{4^2} = \frac{54}{ar(\Delta DEF)} \Rightarrow ar(\Delta DEF) = \frac{54\times16}{9} = 96\ cm^2$

OR

Given, AR = 5 cm, BR = 4 cm and AC =11 cm

We know that the lengths of tangents drawn to the circle from an external point are equal.

Therefore, AR = AQ = 5 cm, BR = BP = 4 cm and

PC = QC = AC – AQ = 11 cm – 5 cm = 6 cm

BC = BP + PC = 4 cm + 6 cm = 10 cm

11. $a = p$, $d = q$ and $n = 10$

$a_{10} = a + (n - 1)d = p + 9q$

OR

As 2x, x + 10, and 3x + 2 are in A.P.

$\Rightarrow 2(x + 10) = 2x + 3x + 2$

$\Rightarrow 2x + 20 = 5x + 2$

$\Rightarrow 3x = 18$

$\Rightarrow x = 6$

12. Let the required polynomial be $ax^2 + bx + c$, and let its zeroes be α and β.

$$\alpha + \beta = \frac{1}{4} = \frac{-b}{a} \text{ and } \alpha\beta = -1 = \frac{-4}{4} = \frac{c}{a}$$

If $a = 4k$, then $b = -k$, $c = -4k$

Therefore, the quadratic polynomial is $k(4x^2 - x - 4)$, where k is a real number

13. LCM of $2^3 \times 3 \times 5$ and $2^4 \times 5 \times 7$ is $3 \times 5 \times 7 \times 2^4 = 1680$.

14. $x^2 + 4x + k = 0$

$\Rightarrow a = 1, b = 4, c = k$

The equation $x^2 + 4x + k = 0$ has real and distinct roots i.e. $b^2 - 4ac > 0$

$\Rightarrow b^2 - 4ac = 4^2 - 4k = 16 - 4k$

$\Rightarrow 16 - 4k > 0$

$\Rightarrow 4k < 16$

$\Rightarrow k < 4$

15. The graph of p(x) intersects the x-axis at only 1 point.

So, the number of zeroes is 1.

16. Given equation is $px^2 + 6x + 1 = 0$

Here, $a = p$, $b = 6$ and $c = 1$

The given equation will have real roots, if $b^2 - 4ac \geq 0$.

$\Rightarrow (6)^2 - 4(p)(1) \geq 0$

$\Rightarrow 36 - 4p \geq 0$

$\Rightarrow 36 \geq 4p$

$\Rightarrow p \leq 9$

Section II

17.

(a) In the first round 1, 3, 5, 7 and 9 numbered triangles are removed.
This means, Rahul is the alternate removing triangles.
In the second round 4 and 8 numbered triangles are removed.
In the third round 6 numbered triangle is removed.
So, 2 numbered triangle will be left in the last.

(b) Removed triangles numbered in sequence are 3, 6, 9, 4, 8, 5, 2 and 7.
So, 1 numbered triangle will be left in the end.

(c) Removed triangles numbered in sequence are 8, 3, 9, 2, 6, 10, 11, 7, 4, 1
So, 5 numbered triangle will be left in the end.

(d) The perimeters of the triangle will follow the below pattern 3, 6, 9, 12, 15, 18, 21, 24 and 27
$\Rightarrow$ They are multiples of 3.

(e) We know that, area of an equilateral triangle = $\frac{\sqrt{3}}{4}(side)^2$

The ratio of the areas of first two triangles whose sides are 3 and 4 is 9: 16
The ratio of the areas of two triangles whose sides are 6 and 8 is 36: 64 = 9: 16.

Hence, they are in proportion as their ratio is same and that is 9: 16.

18.

(a) Angles ∠LKM and ∠JKL are called as Linear Pair of angles.

(b) $m\angle LKM + m\angle JKL = 180°$...... Linear Pair
$\Rightarrow 2x - 15 + m\angle LKM = 180°$
$\Rightarrow m\angle LKM = 195° - 2x$

(c) In △LKM,
$m\angle LKM + m\angle LMK + m\angle KLM = 180°$ ...angle sum property of a triangle
$\Rightarrow 195° - 2x + 50 + x = 180°$
$\Rightarrow x = 65° = m\angle KLM$

(d) $m\angle LKM = 195° - 2x = 195 - 2(65) = 195 - 130 = 65°$
In △LKM, $m\angle LKM = m\angle KLM = 65°$
$\Rightarrow$△LKM is an isosceles triangle.

(e) $m\angle LKJ = 2x - 15 = 2(65) - 15 = 130 - 15 = 115°$

19.

(a) The coordinates of CAVE of DEATH is (5, 3).

(b) The coordinates of THREE PALMS is (6, 4).

(c) The coordinates FOUR CROSS CLIFF and CAVE of DEATH are (2, 3) and (5, 3) respectively.

Distance between them = $\sqrt{(5-2)^2+(3-3)^2} = \sqrt{9} = 3$ units

(d) The distance of SKULL ROCK from x – axis is 5 units.

(e) The mid – point of CAVE of DEATH and THREE PALMS

$$= \left(\frac{5+6}{2}, \frac{3+4}{2}\right) = (5.5, 3.5)$$

20.

(a)

Time (in sec)	No. of students(f)	X	fx
20 – 40	7	30	210
40 – 60	10	50	500
60 – 80	15	70	1050
80 – 100	5	90	450
100 – 120	3	110	330
	Σf = 40		**Σ**fx = 2540

Mean time taken by a student to finish the race = 2540/40 = 63.5 seconds

(b) The modal class is 60 – 80 as it has the highest frequency i.e 15.
Lower limit of the modal class = 60

(c) Mean, Median and Mode are measures of central tendency.

(d)

Time (in sec)	No. of students(f)	cf
20 – 40	7	7
40 – 60	10	17
60 – 80	15	32
80 – 100	5	37
100 – 120	3	40
	N= **Σ**f = 40	

Here N/2 = 40/2 = 20, Median Class = 60 – 80, Modal Class = 60 – 80

Sum of upper limits of median class and modal class = 80 + 80 = 160

(e) Number of students who finished the race within 1 min = 7 + 10 = 17

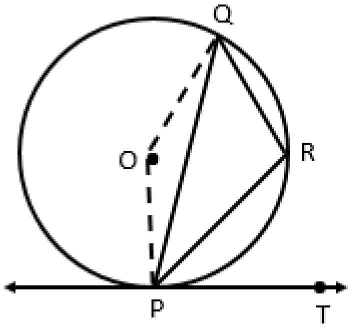
Part B

Section III

21. If 4^n ends with 0, then it must have 5 as a factor.
But, we know that the only prime factor of 4^n is 2.
Also, the fundamental theorem of arithmetic states that the prime factorization of each number is unique.
Hence, 4^n can never end with 0.

22. Since the lengths of tangents from an exterior point to a circle are equal.
Therefore, XP = XQ (tangents from X) ....(i)
AP = AR (tangents from A) ....(ii)
BQ = BR (tangents from B) ....(iii)
Now, XP = XQ
$\Rightarrow$ XA + AP = XB + BQ
$\Rightarrow$ XA + AR = XB + BR [Using (ii) and (iii)]

OR



$m\angle OPT = 90°$ ($\because$ radius is perpendicular to the tangent)

So, $\angle OPQ = \angle OPT - \angle QPT = 90° - 60° = 30°$....(i)

In ΔPOQ, OP = OQ....radius of same circle

$m\angle OPQ = m\angle OQP$.....(angles opposite to equal sides)

$\Rightarrow m\angle OPQ = m\angle OQP = 30°$....from (i)

In ΔPOQ, $m\angle POQ = 180° - (m\angle OPQ + m\angle OQP) = 180° - 60° = 120°$

$2\angle QPT = 2 \times 60° = 120°$ $\because$ angle subtended by an arc

reflex $m\angle POQ = 360° - 120° = 240°$

$m\angle PRQ = \frac{1}{2}$ reflex $\angle POQ = \frac{1}{2} \times 240° = 120°$

$\therefore m\angle PRQ = 120°$

23. Total number of balls in the bag = 3 red + 5 black = 8 balls
Number of total outcomes when a ball is drawn at random = 3 + 5 = 8
Number of favourable outcomes for the red ball = 3

Probability of getting a red ball = P (E) = $\frac{3}{8}$

If $P(\bar{E})$ is the probability of drawing no red ball, then

$P(E) + P(\bar{E}) = 1$

$P(\bar{E}) = 1 - P(E) = 1 - \frac{3}{8} = \frac{5}{8}$

24. According to the question,

Cone:

Radius = r, height = h and volume = V = $\frac{1}{3}\pi r^2 h$

Cylinder:

Radius = r, height = h and volume = V = $\pi r^2 h$

Ratio of volumes = $\dfrac{\pi r^2 h}{\frac{1}{3}\pi r^2 h} = \dfrac{3}{1}$

25. In ΔABC, DE ‖ BC.

Then, by Basic Proportionality Theorem,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{x}{x-2} = \frac{x+2}{x-1}$$

$$\Rightarrow x(x-1) = (x-2)(x+2)$$

$$\Rightarrow x^2 - x = x^2 - 4$$

$$\Rightarrow x = 4$$

OR

The ratio of the areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides.

Let 'a' be the area of smaller triangle and 'A' be the area of the larger triangle.

$$\frac{a}{A} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

$\Rightarrow \dfrac{48}{A} = \dfrac{4}{9}$ ∵ Area of smaller triangle = 48 cm^2

$\Rightarrow A = 108\ cm^2$

26. Radius of the cone = 12 cm and its height = 24 cm

Volume of the cone = $\frac{1}{3}\pi R^2 h = \left(\frac{1}{3} \times \pi \times 12 \times 12 \times 24\right) cm^3 = (48 \times 24)\pi\ cm^3$

Volume of each ball $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 3 \times 3 \times 3 = (36\pi) cm^3$

Number of balls formed $= \dfrac{\text{Volume of solid cone}}{\text{Volume of each ball}} = \dfrac{(48 \times 24)\pi}{36\pi} = 32$

Section IV

27. L.H.S. $= \dfrac{\sec A + \tan A}{\sec A - \tan A}$

$= \dfrac{\sec A + \tan A}{\sec A - \tan A} \times \dfrac{\sec A + \tan A}{\sec A + \tan A}$

$= \dfrac{(\sec A + \tan A)^2}{\sec^2 A - \tan^2 A}$

$= (\sec A + \tan A)^2 \quad (\because \sec^2\theta = 1 + \tan^2\theta \Rightarrow \sec^2\theta - \tan^2\theta = 1)$

$= \left(\dfrac{1}{\cos A} + \dfrac{\sin A}{\cos A}\right)^2$

$= \left(\dfrac{1 + \sin A}{\cos A}\right)^2$

= R.H.S.

OR

$\left(\dfrac{\sin 47^\circ}{\cos 43^\circ}\right)^2 + \left(\dfrac{\cos 43^\circ}{\sin 47^\circ}\right)^2 - 4\cos^2 45^\circ$

$= \left(\dfrac{\sin(90^\circ - 43^\circ)}{\cos 43^\circ}\right)^2 + \left(\dfrac{\cos(90^\circ - 47^\circ)}{\sin 47^\circ}\right) - 4\cos^2 45^\circ$

$= \left(\dfrac{\cos 43^\circ}{\cos 43^\circ}\right)^2 + \left(\dfrac{\sin 47^\circ}{\sin 47^\circ}\right)^2 - 4\left(\dfrac{1}{\sqrt{2}}\right)^2$

$= 1 + 1 - 4 \times \dfrac{1}{2}$

$= 2 - 2$

$= 0$

28. $\dfrac{x}{a} + \dfrac{y}{b} = 2$

$\Rightarrow bx + ay = 2ab$ ... (1)

$ax - by = a^2 - b^2$ ... (2)

Multiplying (1) with a and (2) with b and subtracting, we get

$\cancel{abx} + a^2y = 2a^2b$

$\cancel{abx} - b^2y = a^2b - b^3$

$-\quad + \quad\quad - \quad +$

$y(a^2 + b^2) = a^2b + b^3$

$\Rightarrow y(a^2 + b^2) = b(a^2 + b^2)$

$\Rightarrow y = b$

From (1), $bx + ab = 2ab$

$\Rightarrow bx = ab$

$\Rightarrow x = a$

Hence, $x = a$ and $y = b$.

29. Let $\frac{3}{2\sqrt{5}}$ be a rational number.

$\Rightarrow \frac{3}{2\sqrt{5}} = \frac{a}{b}$, where a and b are co-prime integers and b ≠ 0.

$\Rightarrow \sqrt{5} = \frac{3b}{2a}$

Now, a, b, 2 and 3 are integers.

Therefore, $\frac{3b}{2a}$ is a rational number.

$\Rightarrow \sqrt{5}$ is a rational number.

This is a contradiction as we know that $\sqrt{5}$ is an irrational number.

Therefore, our assumption is wrong.

Hence, $\frac{3}{2\sqrt{5}}$ is an irrational number.

OR

We have $96 = 2^5 \times 3$ and $404 = 2^2 \times 101$

HCF = $2^2 = 4$

HCF × LCM = 96 × 404

$\text{LCM} = \frac{96 \times 404}{\text{HCF}} = \frac{96 \times 404}{4} = 96 \times 101 = 9696$

30. Total no. of cards = 18

$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$

i. No. of favourable outcomes = 7

(Prime nos. in between 1 and 18 are 2, 3, 5, 7, 11, 13, and 17)

P(getting a prime no.) = $\frac{7}{18}$

ii. Factors of 18 are 1, 2, 3, 6, 9, and 18

No. of favourable outcomes = 6

P(getting a factor of 18) = $\frac{6}{18} = \frac{1}{3}$

iii. Numbers divisible by 2 and 3 are 6, 12 and 18

No. of favourable outcomes = 3

P(getting a no. divisible by 2 and 3) = $\frac{3}{18} = \frac{1}{6}$

31. Here it is given that,

$T_{14} = 2(T_8)$

$\Rightarrow a + (14 - 1)d = 2[a + (8 - 1)d]$

$\Rightarrow a + 13d = 2[a + 7d]$

$\Rightarrow a + 13d = 2a + 14d$

$\Rightarrow 13d - 14d = 2a - a$

$\Rightarrow -d = a \quad .... (1)$

Now, it is given that its 6^{th} term is –8.

$T_6 = -8$

$\Rightarrow a + (6 - 1)d = -8$

$\Rightarrow a + 5d = -8$

$\Rightarrow -d + 5d = -8 \quad ... \quad$ [Using (1)]

$\Rightarrow 4d = -8$

$\Rightarrow d = -2$

Substituting this in eq. (1), we get a = 2

Now, the sum of 20 terms,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{20} = \frac{20}{2}[2a + (20-1)d]$$

$$= 10[2(2) + 19(-2)]$$

$$= 10[4 - 38]$$

$$= -340$$

32. Now $2 + \sqrt{3}$ and $2 - \sqrt{3}$ are the two zeroes of the given polynomial

So the product $\left[x - \left(2+\sqrt{3}\right)\right]\left[x - \left(2-\sqrt{3}\right)\right]$ will be a factor of the given polynomial

$$\therefore \left[x - \left(2+\sqrt{3}\right)\right]\left[x - \left(2-\sqrt{3}\right)\right] = (x-2)^2 - \left(\sqrt{3}\right)^2$$

$$= x^2 - 4x + 4 - 3$$

$$= x^2 - 4x + 1$$

let $f(x) = 2x^4 - 9x^3 + 5x^2 + 3x - 1$

and $g(x) = x^2 - 4x + 1$

Find $\frac{f(x)}{g(x)}$.

$$\begin{array}{r|l} & 2x^2 - x - 1 \\ x^2 - 4x + 1 & 2x^4 - 9x^3 + 5x^2 + 3x - 1 \\ & 2x^4 - 8x^3 + 2x^2 \\ & - \quad + \quad - \\ \hline & -x^3 + 3x^2 + 3x \\ & -x^3 + 4x^2 - x \\ & + \quad - \quad + \\ \hline & -x^2 + 4x - 1 \\ & -x^2 + 4x - 1 \\ & + \quad - \quad + \\ \hline & 0 \end{array}$$

$\therefore f(x) = (x^2 - 4x + 1)(2x^2 - x - 1)$

$\therefore 2x^4 - 9x^3 + 5x^2 + 3x - 1 = (x^2 - 4x + 1)(2x^2 - x - 1)$

Hence, the other zeroes of f(x) are the zeroes of the Polynomial $2x^2 - x - 1$.

$\therefore 2x^2 - x - 1 = 2x^2 - 2x + x - 1 = (2x + 1)(x - 1)$

So, $2x^4 - 9x^3 + 5x^2 + 3x - 1 = (x^2 - 4x + 1)(2x^2 - x - 1)$

$$= \left[x - (2+\sqrt{3})\right]\left[x - (2-\sqrt{3})\right](2x + 1)(x - 1)$$

Hence the roots of the Polynomial f(x) are $(2+\sqrt{3}), (2-\sqrt{3}), \frac{-1}{2}$ and 1.

33. Radius of the circle = 14 cm

Central Angle, θ = 60°,

Area of the minor segment

$$= \frac{\theta}{360^\circ} \times \pi r^2 - \frac{1}{2} r^2 \sin\theta$$

$$= \frac{60^\circ}{360^\circ} \times \pi \times 14^2 - \frac{1}{2} \times 14^2 \times \sin 60^\circ$$

$$= \frac{1}{6} \times \frac{22}{7} \times 14 \times 14 - \frac{1}{2} \times 14 \times 14 \times \frac{\sqrt{3}}{2}$$

$$= \frac{22 \times 14}{3} - 49\sqrt{3}$$

$$= \frac{22 \times 14}{3} - \frac{147\sqrt{3}}{3}$$

$$= \frac{308 - 147\sqrt{3}}{3} cm^2$$

Area of the minor segment $= \frac{308 - 147\sqrt{3}}{3} cm^2$

Area of major segment

$$= \pi r^2 - \frac{308 - 147\sqrt{3}}{3} cm^2$$

$$= \frac{22}{7} \times 14 \times 14 - \frac{308 - 147\sqrt{3}}{3}$$

$$= 616 - \frac{308 - 147\sqrt{3}}{3} = 598.1\ cm^2$$

Section V

34. Steps of construction:-

(1) Draw a line segment AB of length 4.6 cm.

(2) At A draw an angle BAY of 60˚.

(3) With centre B and radius 5.1 cm, draw an arc which intersects line AY at point C.

(4) Join BC.

(5) At A draw an acute angle BAX of any measure.

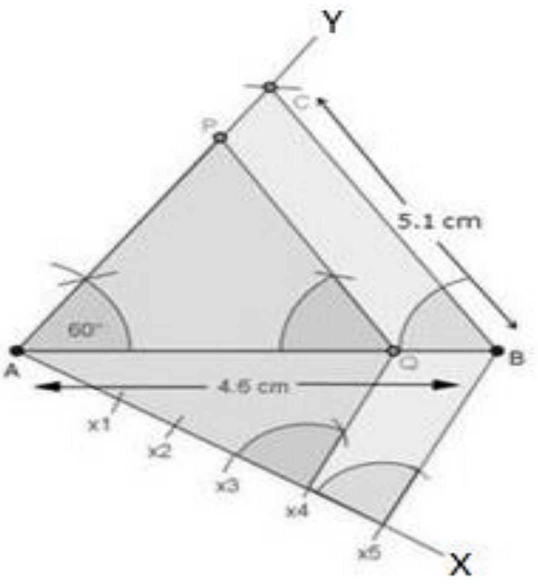
(6) Starting from A, cut 5 equal parts on AX.

(7) Join X_5B.

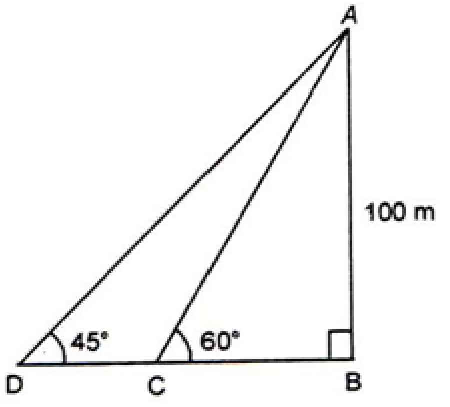
(8) Through X_4, Draw $X_4Q \parallel X_5B$.

(9) Through Q, Draw QP || BC

$\therefore \Delta PAQ \sim \Delta CAB$



35.



Here, the man has covered the distance CD in 2 minutes.

$$\text{Speed} = \frac{\text{Distance}}{\text{time}}$$

Now, in ΔABC,

$$\frac{100}{BC} = \tan 60^{\circ} = \sqrt{3} \Rightarrow BC = \frac{100}{\sqrt{3}} = \frac{100\sqrt{3}}{3}$$

$$\text{In } \Delta ABD, \frac{100}{BD} = \tan 45^{\circ} = 1 \Rightarrow \quad BD = 100$$

$$\therefore CD = BD - BC = \left(100 - \frac{100\sqrt{3}}{3}\right) = 100\left(\frac{3-\sqrt{3}}{3}\right)$$

$$\text{Thus, Speed} = \frac{100\left(\frac{3-\sqrt{3}}{3}\right)}{2} = 50\left(\frac{3-\sqrt{3}}{3}\right) m / min$$

36. Let list price of the book = Rs. x

So, number of books purchased = $\frac{1200}{x}$

And increased price of the book = Rs. (x + 10)

So, number of books purchased = $\frac{1200}{x+10}$

According to condition, if the list price of a book is increased by Rs. 10, then a person can buy 10 less books.

$$\therefore \frac{1200}{x} - \frac{1200}{x+10} = 10$$

$$\therefore (1200)\left[\frac{1}{x} - \frac{1}{x+10}\right] = 10$$

$$\therefore (1200)\left[\frac{x+10-x}{x(x+10)}\right] = 10$$

$\therefore 1200 = x(x + 10)$

$\therefore x^2 + 10x - 1200 = 0$

$\therefore (x + 40)(x - 30) = 0$

$\therefore x = -40$ or $x = 30$

But x is the list price of the book and hence can't be negative.

Therefore, the original list price of the book is Rs. 30.

OR

Let the speed of the stream be x km/ hr.

Here, the speed of the motor boat is 15km/ hr in still water.

∴ Speed downstream = (15 + x) km/hr and

Speed upstream = (15 – x) km/hr

A boat goes 30 km downstream and comes back,

∴ Distance covered in downstream = 30 km and

Distance covered in upstream = 30 km

Total time taken by A boat = 4 hrs 30 mins = $4\frac{30}{60}$ hrs = $\frac{9}{2}$ hrs

$$\therefore \left(\frac{30}{15+x}\right)+\left(\frac{30}{15-x}\right)=\frac{9}{2}$$

Taking L.C.M as (15 + x) (15 – x)

$$\therefore \frac{30(15-x)+30(15+x)}{(15+x)(15-x)}=\frac{9}{2}$$

$$\therefore 30(15-x+15+x)=\frac{9}{2}(15+x)(15-x)$$

$$\therefore 30 \times 30=\frac{9}{2}(15^2-x^2)$$

$$\therefore \frac{900 \times 2}{9}=225-x^2$$

$\therefore 200 = 225 - x^2$

$\therefore x^2 = 25$

$\therefore x = 5$ or -5

Speed is always positive,

$\therefore x = 5$

Therefore, the speed of stream is 5 km/hr.